Day 20

Kalman Filter

Plant or Process Model

▶ describes how the system state changes as a function of time, control input, and noise

$$x_{k+1} = \Phi x_k + \Gamma u_k + v_k$$

  ▶ x_k state at time k
  ▶ u_k control inputs at time k
  ▶ v_k process noise at time k
  ▶ Φ state transition model or matrix
  ▶ Γ control-input model or matrix
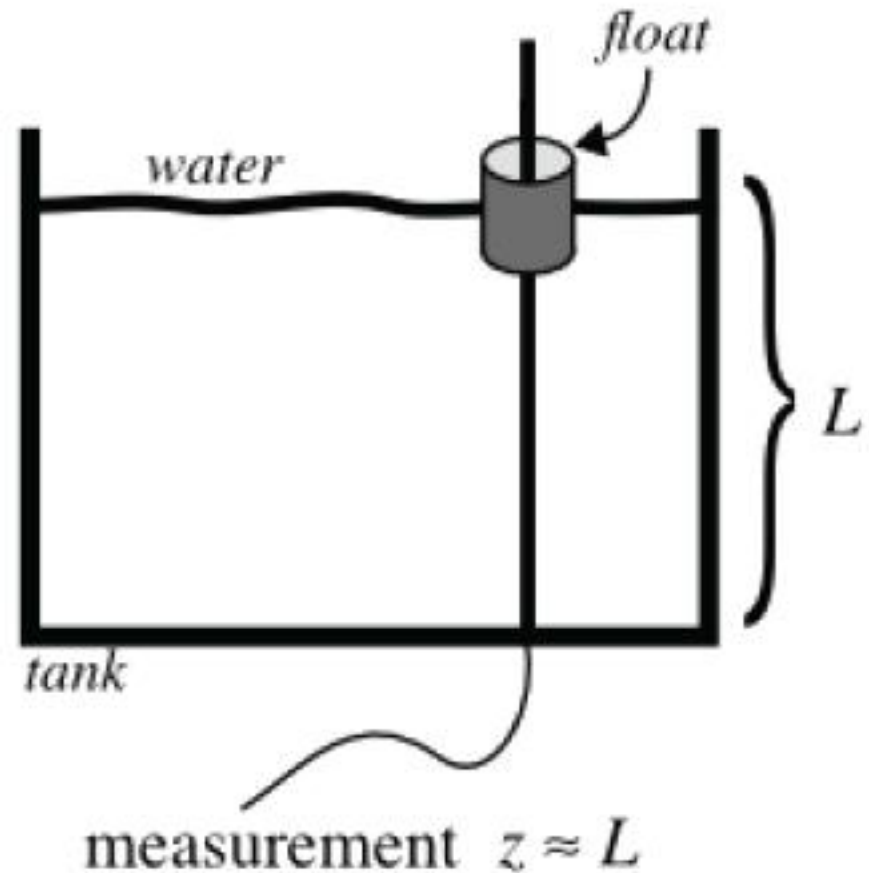
Measurement Model

▸ describes how sensor measurements vary as a function of the system state
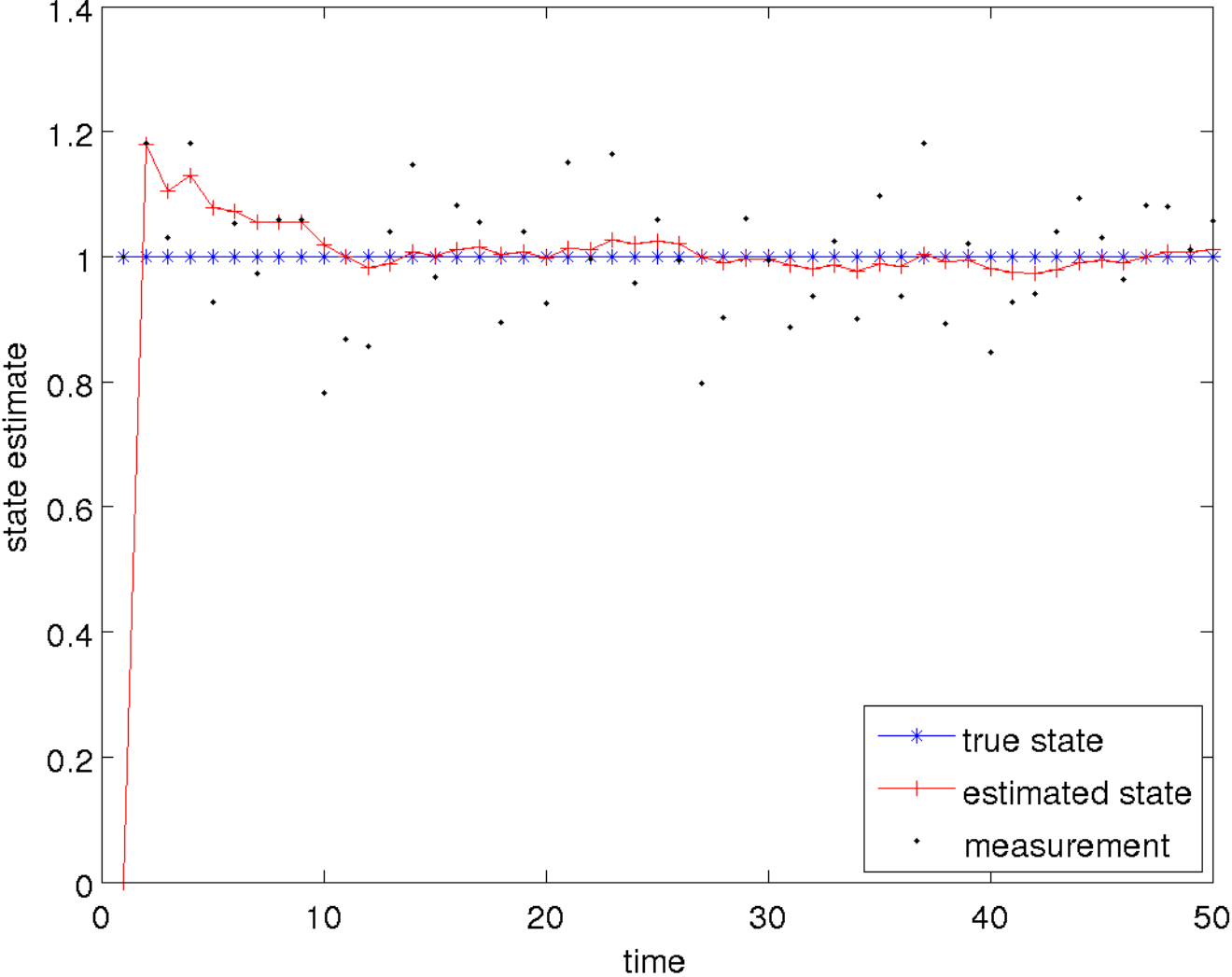
$$z_k = \Lambda x_k + w_k$$

- ▸ z_k sensor measurement at time k
- ▸ w_k sensor noise at time k
- ▸ Λ observation model or matrix

Tank of Water

▸ estimate the level of water in the tank; the water could be
  ▸ static, filling, or emptying
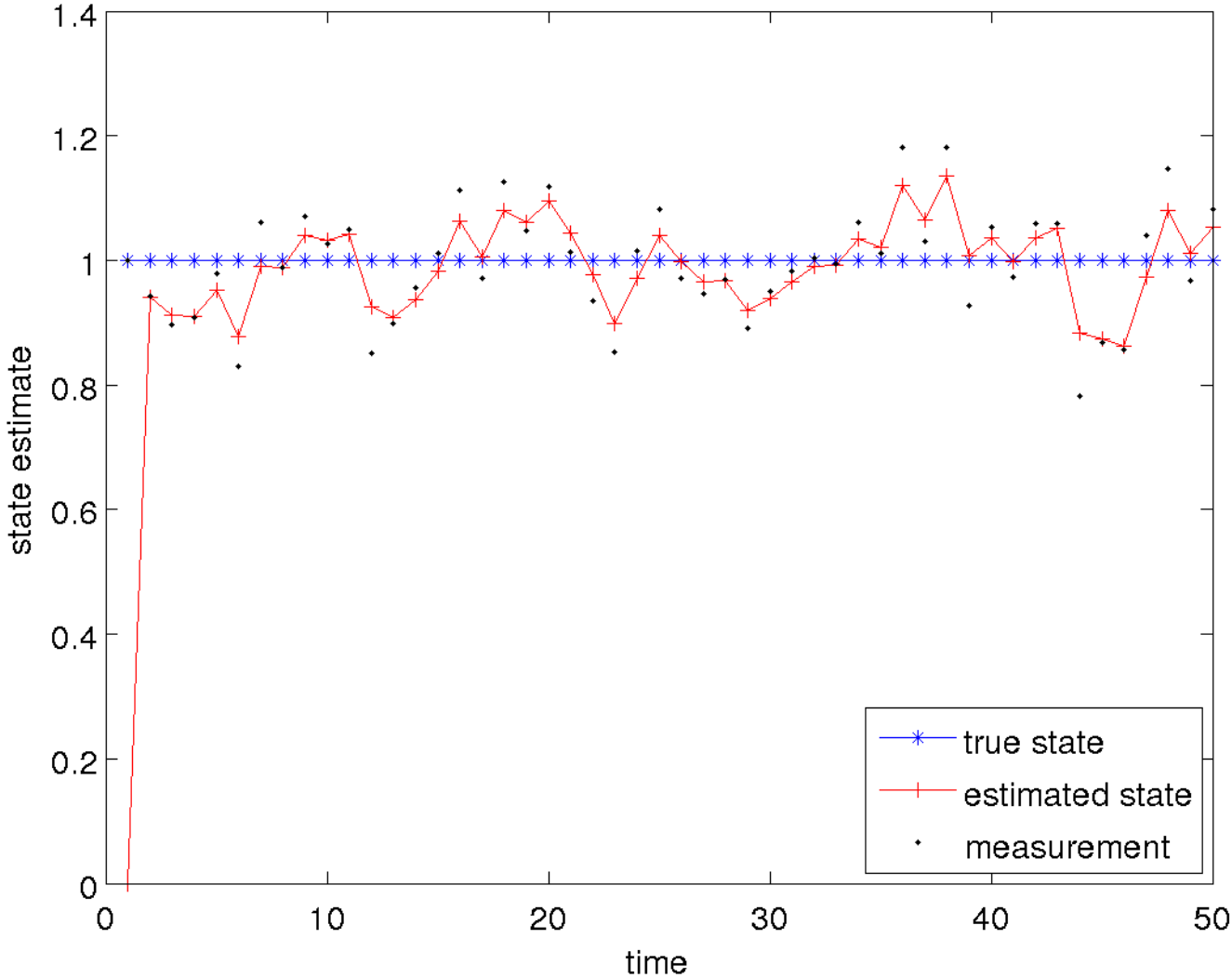 ▸ not sloshing or sloshing

Tank of Water: Static and Not Sloshing

Tank of Water: Static and Not Sloshing

▸ notice that in this case the Kalman filter tends towards estimating a constant level because the plant noise covariance is small compared to the measurement noise covariance

  ▸ the estimated state is much smoother than the measurements

▸ what happens if we increase the plant noise covariance?
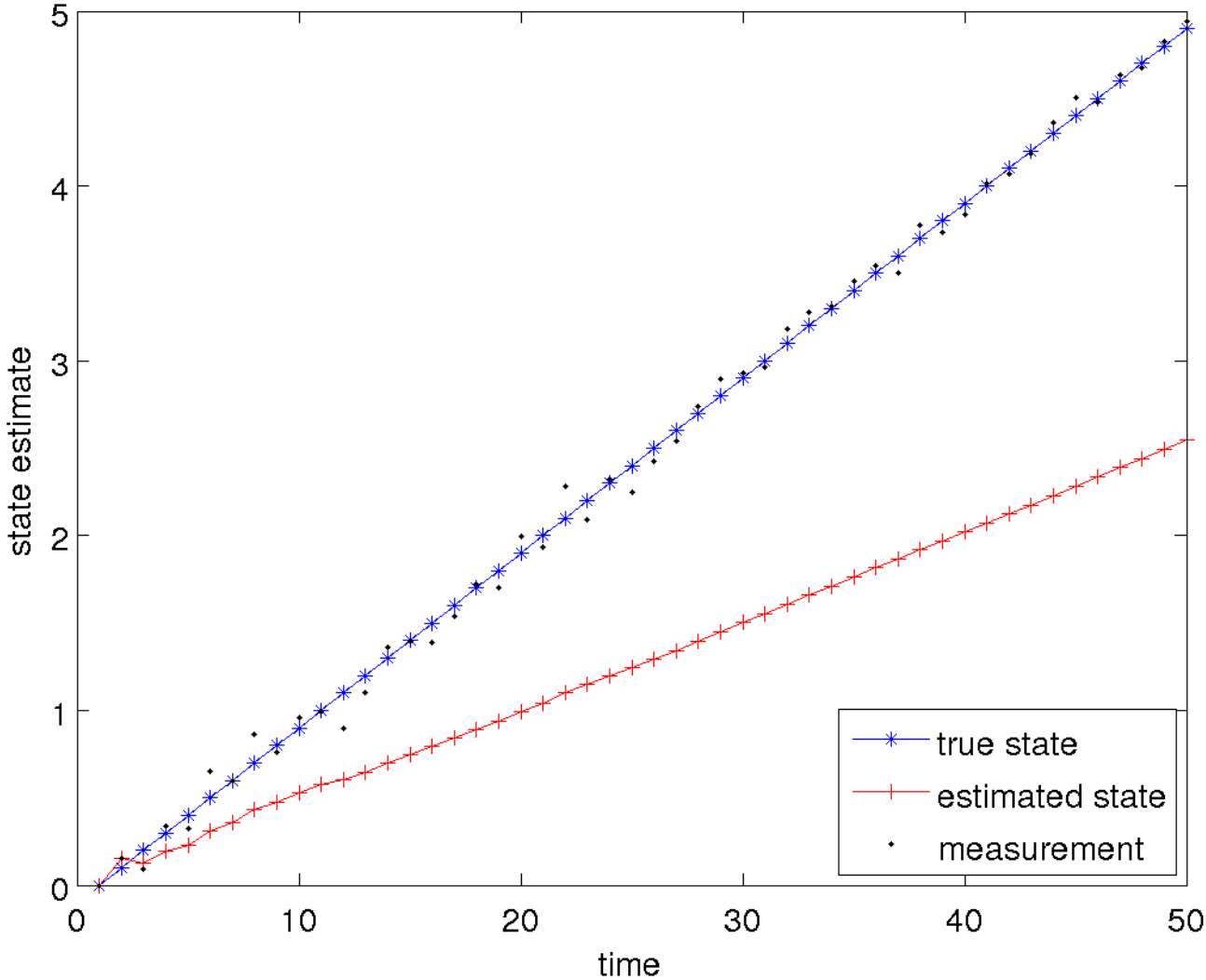
Tank of Water: Filling and Not Sloshing

Tank of Water: Static and Not Sloshing

▸ notice that in this case the Kalman filter tends towards estimating values that are closer to the measurements

▸ increasing the plant noise covariance causes the filter to place more weight on the measurements

Tank of Water: Filling and not Sloshing

‣ suppose the true situation is that the tank is filling at a constant rate but we use the static tank plant model

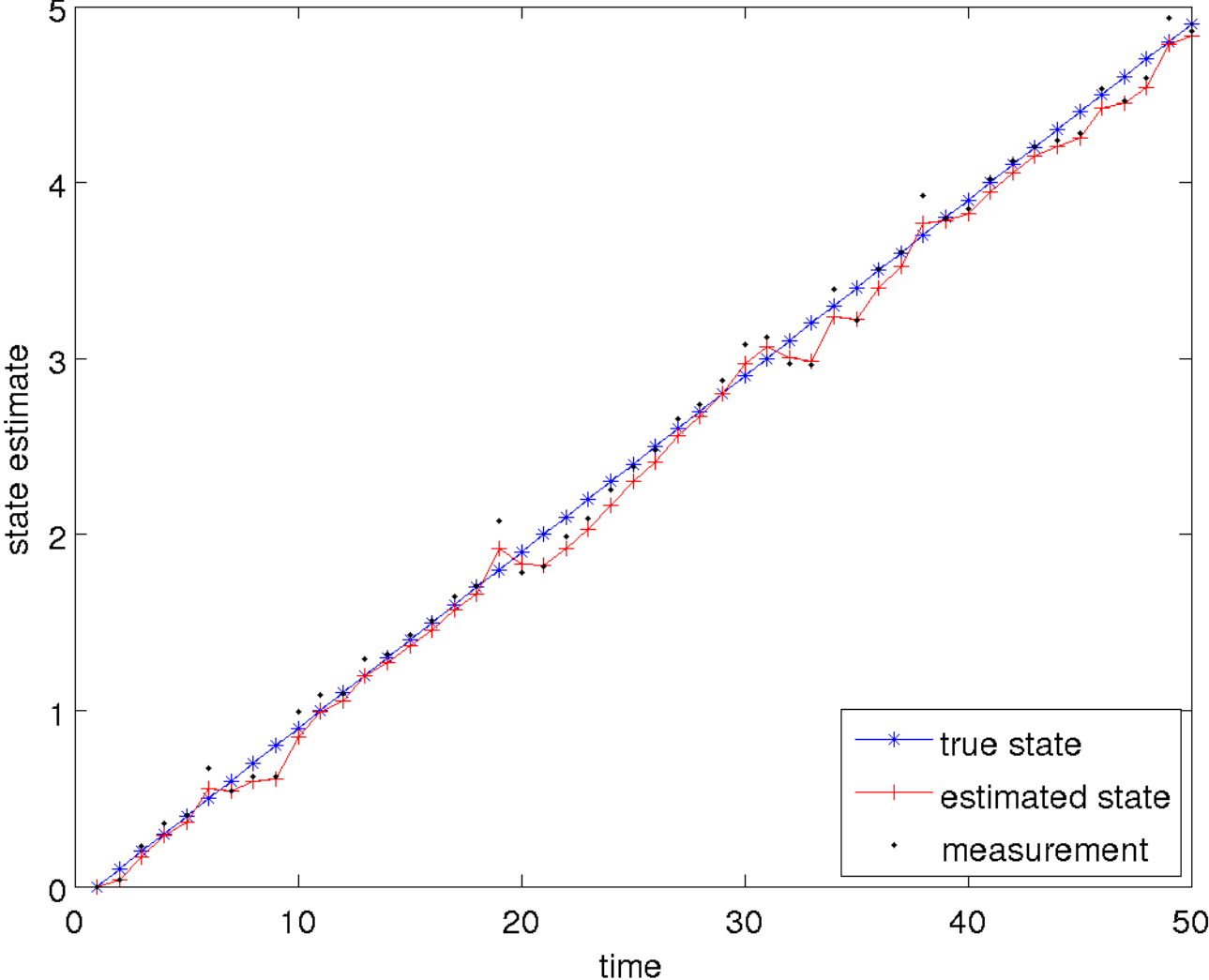   ‣ i.e., we have a plant model that does not accurately model the state transition

Tank of Water: Filling and not Sloshing

Tank of Water: Filling and not Sloshing

- notice that in this case the estimated state trails behind the true level
  - estimated state has a much greater error than the noisy measurements
- if the plant model does not accurately model reality than you can expect poor results
  - however, increasing the plant noise covariance will allow the filter to weight the measurements more heavily in the estimation…
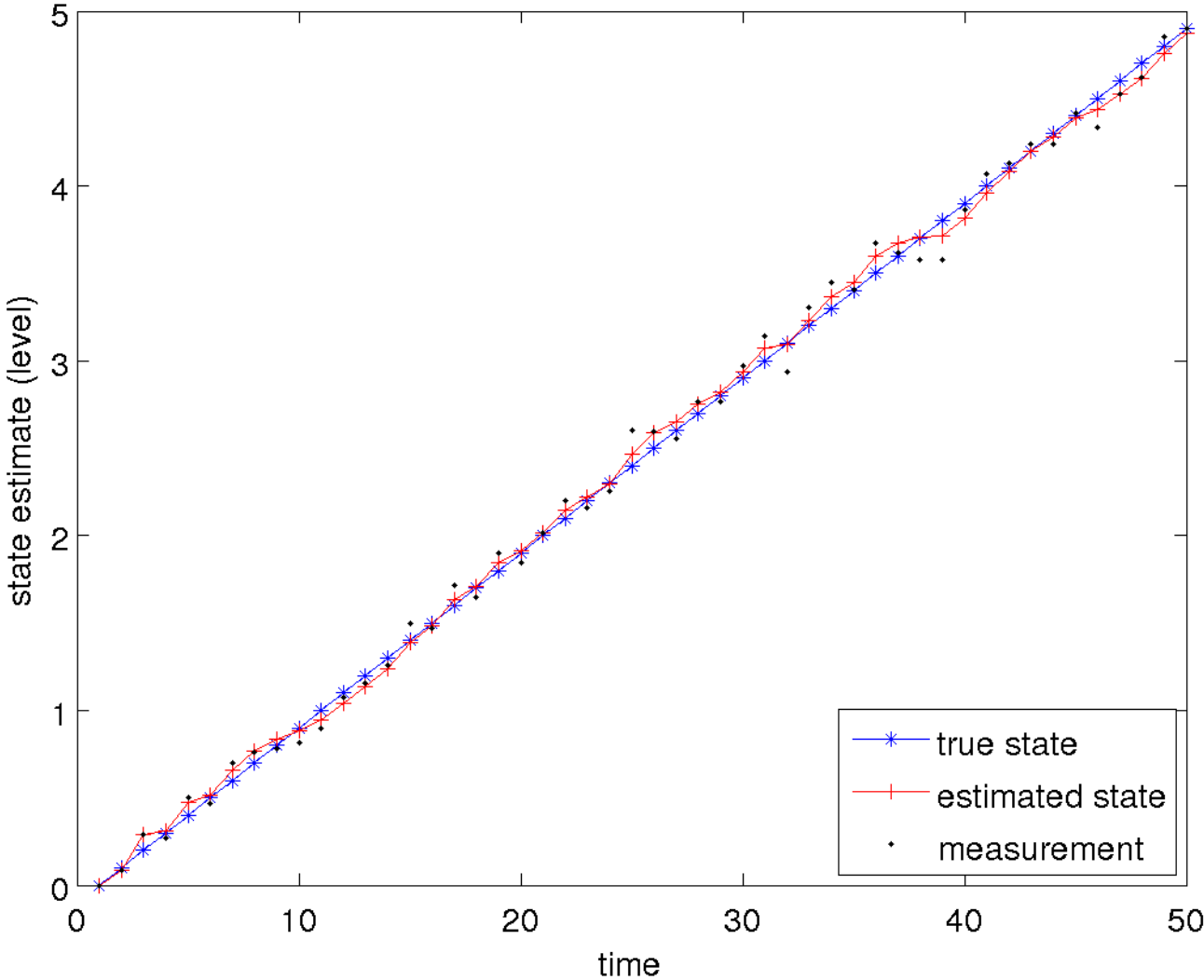
Tank of Water: Filling and not Sloshing

Tank of Water: Filling and not Sloshing

▸ it is not clear if we have gained anything in this case

  ▸ the estimated state is reasonable but it is not clear if it is more accurate than the measurements

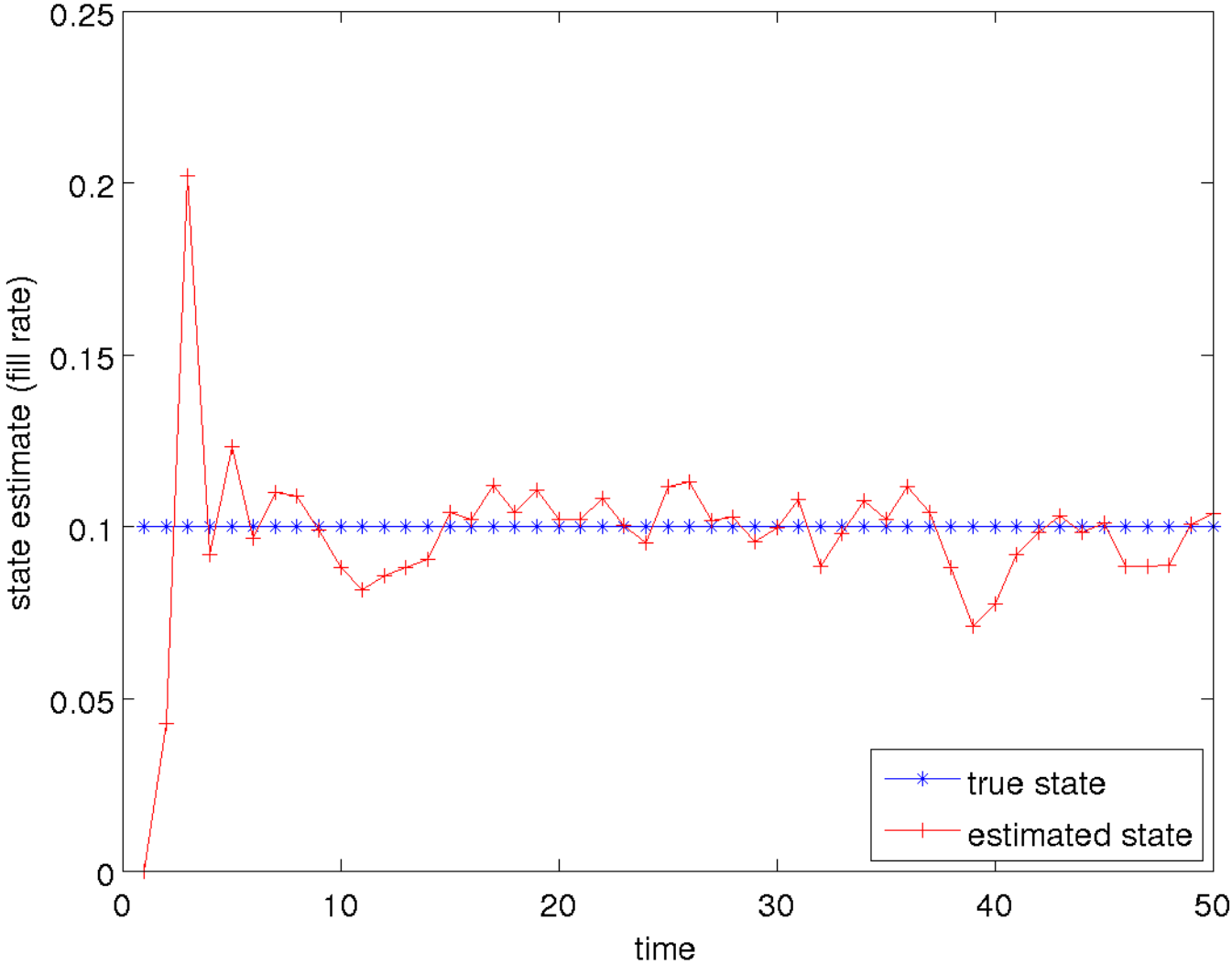▸ what happens if we change the plant model to more accurately reflect the filling?

Tank of Water: Filling and not Sloshing

Tank of Water: Filling and not Sloshing

▸ notice that the estimated state is more accurate and smoother than the measurements

▸ what about the filling rate?
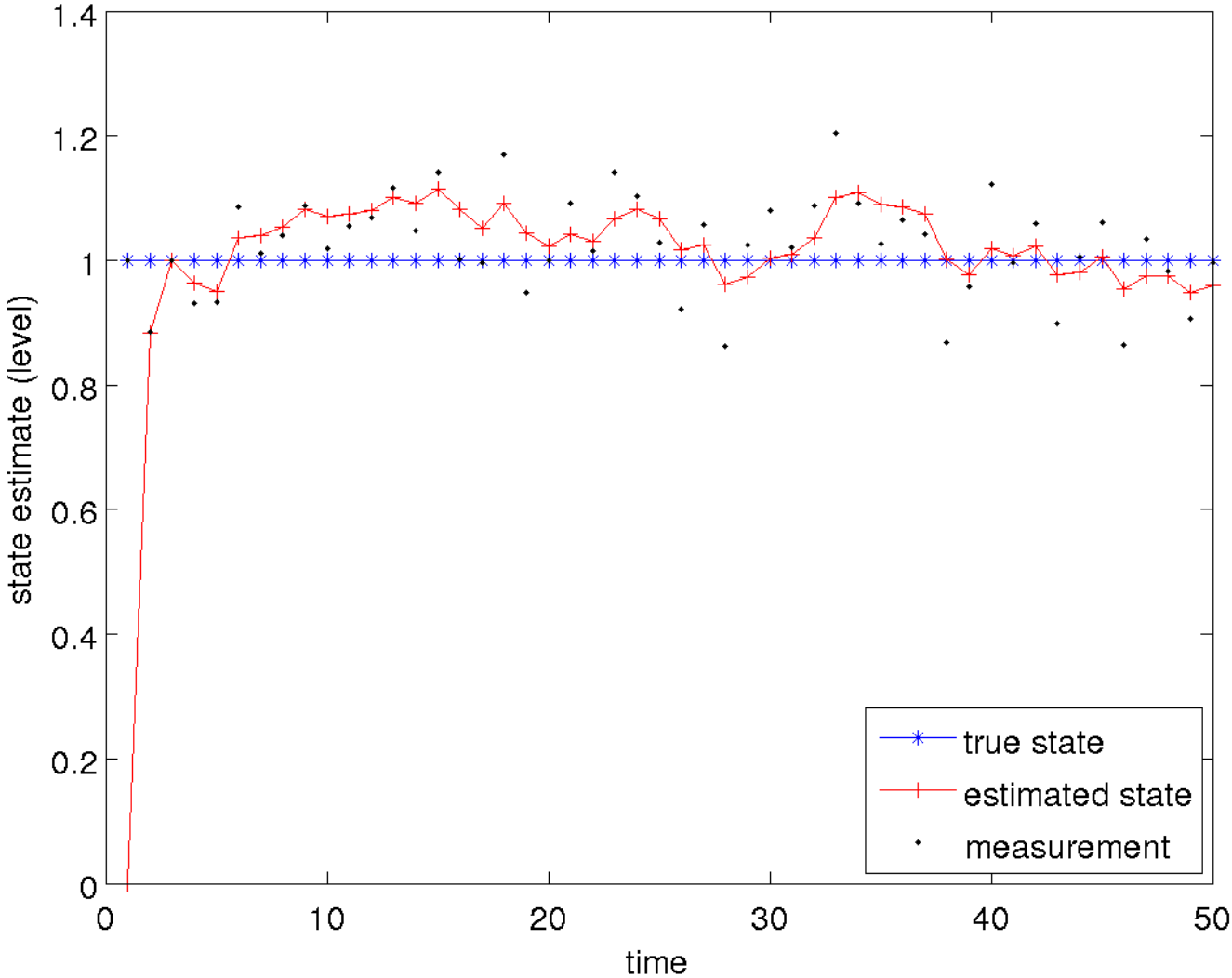
Tank of Water: Filling and not Sloshing

Tank of Water: Filling and not Sloshing

- notice that the estimated filling rate seems to jump more than the estimated level

  - this should not be surprising as we never actually measure the filling rate directly

    - it is being inferred from the measured level (which is quite noisy)

Tank of Water: Static and not Sloshing

▸ can we trick the filter by using the filling plant model when the level is static?

   ▸ hopefully not, as the filter should converge to a fill rate of zero!

Tank of Water: Static and not Sloshing

Tank of Water: Static and not Sloshing